SOLVING QUADRATICS BY COMPLETING THE SQUARE #24

If $(x - a)^2 = b$, then $(x - a) = \pm\sqrt{b}$ and $x = a \pm \sqrt{b}$. This means that if we have a perfect square on one side of an equation we can remove it by taking the square root of each side and then solving in the usual way. Completing the square is a method to transform a quadratic equation into this sometimes more usable form. Also see the textbook, pages 436-38 and 444.

Example 1

Solve $x^2-10x + 22 = 0$

Isolate the constant.

$$x^2 - 10x = -22$$

We need to make x^2-10x into a perfect square. Taking half the x coefficient and squaring it will accomplish this.

$$(x^2 - 10x + ?) = -22, \qquad ? = \left(\frac{-10}{2}\right)^2 = 25$$

The 25 that was put into the parenthesis must be compensated for by adding 25 to the other side so that the equation remains balanced.

$$(x^2 - 10x + 25) = -22 + 25$$

Factor and simplify.

$$(x - 5)^2 = 3$$
$$x - 5 = \pm\sqrt{3}$$
$$x = 5 \pm \sqrt{3}$$

Example 2

Solve $x^2+ 5x + 2 = 0$

Isolate the 2.

$$x^2+ 5x = -2$$

We need to make $x^2+ 5x$ into a perfect square. Again, taking half of the x coefficient and squaring the result will accomplish the task.

$$(x^2+ 5x + ?) = -2, \qquad ? = \left(\frac{5}{2}\right)^2 = \frac{25}{4}$$

The $\frac{25}{4}$ that was put into the parenthesis must be compensated for by adding $\frac{25}{4}$ to each side (then factoring and simplifying).

$$(x^2+ 5x + \tfrac{25}{4}) = -2 + \tfrac{25}{4}$$

$$(x + \tfrac{5}{2})^2 = \tfrac{-8}{4} + \tfrac{25}{4} = \tfrac{17}{4}$$

Solve the equation as usual.

$$(x + \tfrac{5}{2}) = \pm\sqrt{\tfrac{17}{4}} = \pm \tfrac{\sqrt{17}}{2}$$

$$x = \tfrac{-5}{2} \pm \tfrac{\sqrt{17}}{2} \text{ or } \tfrac{-5\pm\sqrt{17}}{2}$$

Solve the following quadratics by completing the square.

1. $x^2 - 2x - 20 = 0$
2. $x^2 - 8x - 10 = 0$
3. $x^2 + 12x = -3$
4. $y^2 + 10y = -7$
5. $x^2 + 4x - 3 = 0$
6. $y^2 + 14y + 5 = 0$
7. $y^2 - 3y = -2$
8. $m^2 - 9m - 4 = 3$
9. $w^2 + w = 5$
10. $x^2 - x - 20 = 0$
11. $4x^2 + 4x + 1 = 5$
12. $2x^2 + 12x = 18$

Answers

1. $1 \pm \sqrt{21}$
2. $4 \pm \sqrt{26}$
3. $-6 \pm \sqrt{33}$
4. $-5 \pm \sqrt{18}$
5. $-2 \pm \sqrt{7}$
6. $-7 \pm \sqrt{44}$
7. 2, 1
8. $\frac{9 \pm \sqrt{109}}{2}$
9. $\frac{9 \pm \sqrt{109}}{2}$
10. 5, -4
11. $\frac{-1 \pm \sqrt{5}}{2}$
12. $-3 \pm \sqrt{18}$